where P and Σ are $2n \times 2n$ symmetric matrices, and $g=(g_1,g_2,\ldots,g_p)^T$.

Outline of Proof: The structure of the proof is very similar to that used for the general optimal output feedback problem.[7] (It should be noted that the proof given in Ref. 7 needs slight modification in light of Ref. 8, although the end result is the same.) The only difference is that the derivative of the Hamiltonian with respect to the vector g (rather than the matrix G) is equated to zero. The following easily proved properties of the matrix trace are used (G is a diagonal matrix, α and β are square matrices of compatible dimension).

$$\frac{\partial}{\partial g}\mathrm{tr}[G\alpha\, G\beta] = \frac{\partial}{\partial g}[g^T(\alpha * \beta)g] = \{\alpha*\beta + (\alpha*\beta)^T\}g \tag{21}$$

$$\frac{\partial}{\partial g}\mathrm{tr}[G\alpha] = \Delta(\alpha) \tag{22}$$

The fact that $C=B^T$ is also used.

It should be noted that, as in the case of the general optimal output feedback problem, the theorem does not guarantee the existence of a g that will make the system asymptotically stable, although the necessary conditions assume the existence. Indeed, the performance function of Eq. (15) will be meaningful only if such a g exists.

The optimal gain vector g may be computed using the algorithm given in Ref. 7, or using a numerical minimization method (such as Davidon-Fletcher-Powell). The algorithm of Ref.7 involves iteratively solving Eqs. (18-20). That is, assuming an initial stable g, Eqs. (19) and (20) are solved for P and Σ, and Eq. (18) is solved to obtain the next value of g, and so forth. Convergence of the algorithm has not been proven, although it has generally been found to converge[7] in the case of a nondiagonal gain matrix.

Concluding Remarks

Necessary conditions are obtained for minimizing a quadratic performance function under the framework of the member damper concept. Knowledge of noise covariances is used in the design. The method presented offers a systematic approach to the design of a class of controllers for enhancing structural damping in large space structures. This type of controller has significant potential if used in conjunction with a reduced-order optimal controller that is designed to control rigid-body modes and some selected structural modes.

References

[1] Balas, M.J., "Feedback Control of Flexible Systems," *IEEE Transactions Automatic Control,* Vol. AC-23, Aug. 1978, pp. 673-679.

[2] Joshi, S.M. and Groom, N.J., "Design of Reduced-Order Controllers for Large Flexible Space Structures," *Proceedings of 16th Annular Allerton Conference on Communication, Control, and Computing,* Oct. 4-6, 1978, pp. 780-791.

[3] Joshi, S.M. and Groom, N.J., "Stability Bounds for the Control of Large Space Structures," *Journal of Guidance and Control,* Vol. 2, July-Aug. 1979, pp. 349-351.

[4] Canavin, J.R., "Control Technology for Large Space Structures," AIAA Paper 78-1691, presented at AIAA Conference on Large Space Platforms, Los Angeles, Calif., Sept. 1978.

[5] Balas, M.J., "Direct Velocity Feedback Control of Large Space Structures," *Journal of Guidance and Control,* Vol. 2, May-June 1979, pp. 252-253.

[6] Strunce, R.R. and Henderson, T.C., "Application of Modern Modal Controller Design to a Large Space Structure," Paper presented at Second VPI&SU Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Va., June 1979.

[7] Joshi, S.M., "Design of Optimal Partial State Feedback Controllers for Linear Systems in Stochastic Environments," *Proceedings of 1975 IEEE Southeastcon,* Charlotte, N.C., April 1975.

[8] Walsh, P.M., "On Symmetric Matrices and the Matrix Minimum Principle," *IEEE Transactions, Automatic Control,* Vol. AC-22, Dec. 1977, pp. 995-996.

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Passive Dissipation of Energy in Large Space Structures

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Introduction

STRUCTURAL damping models currently used in the dynamical analysis of flexible spacecraft frequently lack fidelity. Yet, an accurate assessment of energy dissipation is often crucial in the stabilization and control of these vehicles. To cite two examples, consider the stability of a dual-spin spacecraft with flexible appendages or the potential instabilities in the active control of flexible spacecraft.

Modern structural analysis techniques have tended to focus on the "stiffness matrix" $\boldsymbol{K}$ and the "system inertia matrix" $\boldsymbol{M}$, thereby generating structural-dynamical models of the form

$$M\ddot{q} + Kq = f(t) \tag{1}$$

where q contains suitable generalized coordinates, and f contains the generalized inputs. A notable omission in Eq. (1) is a mechanism for energy dissipation. A common device to remedy this omission is the addition of a linear-viscous damping term $D\dot{q}$. Hence, the "improved" system is

$$M\ddot{q} + D\dot{q} + Kq = f(t) \tag{2}$$

Very little is said in the literature concerning how to calculate $\boldsymbol{D}$. This is not surprising because none of the expected forms of dissipation (including material damping, structural damping, hysteretic damping, stiction, Coulomb damping, freeplay at bolts, rivets, or joints) is in fact linear-viscous damping.

The next step is often to find the system modes for Eq. (2). This requires the simultaneous diagonalization of $\boldsymbol{M}$, $\boldsymbol{D}$, and $\boldsymbol{K}$, which is not, in general, possible. As a mathematical curiosity, this triple diagonalization is possible if $\boldsymbol{D}$ is a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$ (this condition is sufficient, but not necessary). However, the author knows of no physical justification for this assumption. Denoting by $\boldsymbol{T}$ the transformation that diagonalizes $\boldsymbol{M}$ and $\boldsymbol{K}$, so that

$$T^TMT = 1 \qquad T^TKT = \omega^2 \tag{3}$$

where $\boldsymbol{1}$ is the unit matrix, and ω is a diagonal matrix of (undamped) natural frequencies ω_α, the change of variables

$$q = T\eta \tag{4}$$

converts Eq. (2) to

$$\ddot{\eta} + \hat{D}\dot{\eta} + \omega^2\eta = \hat{f}(t) \tag{5}$$

where $\hat{f} \triangleq T^Tf$ and $\hat{D} \triangleq T^TDT$, with $\hat{D}$ generally having off-diagonal elements. In the crusade for uncoupled modal equations, the off-diagonal terms $\hat{d}_{\alpha\beta}$ $(\alpha \neq \beta)$ are often set to zero.

Received Nov. 19, 1979; revision received Feb. 19, 1980.

Index categories: Spacecraft Dynamics and Control; Structural Dynamics.

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The remaining (diagonal) terms are conventionally expressed in terms of modal damping factors ζ_α, as follows:

$$\hat{d}_{\alpha\alpha} = 2\zeta_\alpha \omega_\alpha \tag{6}$$

What is left in this "theory of structural damping" is to choose the ζ_α. Because Eq. (6) has been arrived at by a nonstrict analysis, there is no real basis (except perhaps experiments) for choosing the ζ_α. In the absence of such a basis, the further assumption

$$\zeta_\alpha = \zeta \qquad (\text{all} \quad \alpha) \tag{7}$$

is often employed. All the characteristics of structural damping have thus been reduced to a single parameter ζ.

It remains to choose ζ. The word is "choose," not "calculate." It is typically chosen based on expectations for the general type of structure under consideration. For a structure with "tight" joints, one may think ζ to be ~ 0.001, whereas for a structure with "looser" joints, ζ may be 0.01, or even larger.

Recent developments in damping treatments[1,2] make it clear that damping factors of 1% or even 10% are achievable without undue weight penalties. Moreover, one senses that these practical developments have only just begun, with futuristic special-purpose materials yet to be designed, perhaps involving composite materials, with damping level as an inherent manufacturing specification. From this perspective, the current emphasis on sophisticated control-systems analysis for these large space structures takes on a slightly new meaning.[3] Instead of being given an unalterable structure to control, the control-systems designer must work iteratively with the structural analyst, the former identifying the structural chracteristics that are problematical for controller design, and the latter making appropriate structural modifications. One of the most promising modifications is to exploit the possibilities for passive energy dissipation. This process requires reliable damping models.

Analysis

In view of the importance of energy dissipation mentioned in the first paragraph, the procedure outlined briefly in Eqs. (2-7) is not altogether satisfactory for some applications. The purpose of this Note is to suggest a more accurate and fruitful approach in which the modeling of dissipation is done in the frequency domain, where more accurate damping models are available.

It is a premise in the sequel that the response of the structure to sinusoidal excitation at frequency ω is known for all ω. For the undamped system [Eq. (1)], for example, if $f(t) = f_0 e^{i\omega t}$, then

$$q(t) = (K - \omega^2 M)^{-1} f_0 e^{i\omega t} \tag{8}$$

Let us denote the Fourier transform of $\phi(t)$ by $\mathfrak{F}[\phi]$. Thus,

$$\mathfrak{F}[\phi] = \int_{-\infty}^{\infty} \phi(t) e^{-i\omega t} \mathrm{d}t \tag{9}$$

and the inverse transform is

$$\phi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathfrak{F}[\phi] e^{i\omega t} \mathrm{d}\omega \tag{10}$$

It is known[4] that the response of a linear system to a general input is known once the sinusoidal response is known at all ω. For the undamped system, for example, the response to a general input $f(t)$ is found from the inverse Fourier transform of

$$\mathfrak{F}[q] = (K - \omega^2 M)^{-1} \mathfrak{F}[f] \tag{11}$$

Or, in terms of modal coordinates,

$$\mathfrak{F}[\eta_\alpha] = \mathfrak{F}[\hat{f}_\alpha] / (\omega_\alpha^2 - \omega^2)$$

For the viscously damped model of Eq. (2), the equivalent result is, of course,

$$\mathfrak{F}[q] = (K - \omega^2 M + i\omega D)^{-1} \mathfrak{F}[f] \tag{12}$$

where again $q(t)$ is the inverse Fourier transform of $\mathfrak{F}[q]$. In the frequency domain, however, we can do considerably better than Eq. (12). It is known[5] that for material damping, the "stiffness" is frequency dependent. Thus, $K = K(\omega)$ for real materials. Furthermore, in place of the fictitious viscous damping term in Eq. (12), $i\omega D$, we may write $iN(\omega)$, where $N(\omega)$ is based on "loss factors" that are also frequency dependent. The same remarks apply to complex structures.[6] Using this approach, the response of the structure to a general input $f(t)$ is

$$\mathfrak{F}[q] = [K(\omega) - \omega^2 M + iN(\omega)]^{-1} \mathfrak{F}[f] \tag{13}$$

Unlike Eq. (12), which corresponds to the differential equation (2), the frequency-domain model of Eq. (13) has no corresponding differential equation in the time domain. Although this is a loss, a considerable improvement in model accuracy has been gained.

Some brief comments are in order concerning how to obtain the matrices M, $K(\omega)$, and $N(\omega)$. The system inertia matrix M is calculated in the standard manner, using the finite-element method, for example, or otherwise. For K and N, however, the usual methods are not adapted to finding the frequency dependence. Ideally, one might apply a theory of material damping, a theory of joint dissipation and whatever other theories were relevant to the structure at hand. Another approach,[2,7] in which measurements are used at a later stage in the analysis, consists of subjecting suitably chosen small portions of the structure to a frequency-response experiment and inferring the overall properties of the structure by further analysis. Perhaps the most powerful such technique is to extend the finite-element method[7] to calculate $K(\omega)$ and $N(\omega)$ from material properties. As a further alternative, one can visualize vibration testing in space in which the system response matrix $H(\omega)$ is measured. The response of the space structure to a general (nonsinusoidal) input $f(t)$ is then calculated from

$$\mathfrak{F}[q] = H(\omega) \mathfrak{F}[f] \tag{14}$$

Both amplitude and phase information is contained in the complex function H. Thus, analytical models may be viewed as a means for calculating $H(\omega)$. For example,

$$H(\omega) = [K(\omega) - \omega^2 M + iN(\omega)]^{-1} \tag{15}$$

for the model of Eq. (13). The only assumptions in Eq. (14) are that the structure is linear and time-invariant.

The model represented by $H(\omega)$ may be used in either the frequency or the time domain. Some linear control theories are based on frequency-response functions (Nyquist criterion, Bode plots, etc.), while others are based in the time domain for which the impulse response matrix $Q(t)$ is readily determined from a fast Fourier transform:

$$Q(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega t} \mathrm{d}\omega \tag{16}$$

For example, the response to a control input $f(t)$ is

$$q(t) = \int_0^{\infty} Q(\tau) f(t-\tau) \mathrm{d}\tau \tag{17}$$

In particular, the transient response to initial conditions can be obtained by setting $f(t) = f_0\delta(t)$, for which

$$q(t) = Q(t) f_0 \tag{18}$$

Evidently, the response to initial conditions $q(0)$ is

$$q(t) = Q(t) Q^{-1}(0) q(0) \tag{19}$$

and thus a knowledge of the frequency response $H(\omega)$ provides the key to the transient response as well.

It should be observed that $Q(t)$ is a real causal function. (By "causal function" we mean $Q(t) = 0$ for $t < 0$.) Causality has been assumed in Eq. (17). It is not obvious that $Q(t)$, computed from $H(\omega)$ according to Eq. (16), with $H(\omega)$ in turn calculated from Eq. (15), where $K(\omega)$ and $N(\omega)$ are based on laboratory measurements on small elements, will actually be real and causal. Engineering common sense dictates that the more accurately these procedures are carried out, the more nearly real and causal $Q(t)$ will be. Assuming these properties are closely approached, one may feel justified in guaranteeing them by minor adjustments to $H(\omega)$. To take realness first, let

$$H(\omega) = R(\omega) + iI(\omega) \tag{20}$$

where R and I are real functions of ω. Then, for real $Q(t)$,

$$R(-\omega) = R(\omega) \qquad I(-\omega) = -I(\omega) \tag{21}$$

In other words, R is even and I is odd with respect to ω. This suggests a "correction for realness" as follows:

$$\tfrac{1}{2}[R(\omega) + R(-\omega)] \rightarrow R(\omega) \qquad \tfrac{1}{2}[I(\omega) - I(-\omega)] \rightarrow I(\omega) \tag{22}$$

Similarly, for $H(\omega)$ to correspond to a causal function, $R(\omega)$ and $I(\omega)$ must contain the same information. They satisfy the Hilbert transforms[4]:

$$I(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{R(\nu)}{\omega - \nu}\,d\nu \qquad R(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{I(\nu)}{\omega - \nu}\,d\nu \tag{23}$$

This suggests a "correction for causality" as follows

$$\tfrac{1}{2}H(\omega) - \frac{i}{2}\left[\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H(\nu)}{\omega - \nu}\,d\nu\right] \rightarrow H(\omega) \tag{24}$$

which "touches up" the $H(\omega)$ function so that it corresponds to a causal $Q(t)$. Of course, strict realness and causality are not absolutely essential. The above "corrections" need to be calculated only once for each structure, and apply for all inputs. But, if it is desired to avoid them, it is perhaps better to have a slightly nonreal or slightly noncausal function $Q(t)$ with damping accurately modeled, than a strictly real and causal $Q(t)$ with the model otherwise highly suspect.

Conclusion

Current practice in the dynamical modeling of large space structures is to create accurate M and K matrices, using sophisticated analysis techniques, and then to treat energy dissipation in an ad hoc and rather inaccurate fashion. It is possible to eliminate this disparity by adopting a frequency-domain viewpoint in which frequency-response theory and test results are incorporated. Two areas of concern have been examined, realness and causality, and some suggestions made for coping with them.

References

[1] Nashif, A.D., "Material Property Measurements and Data Reduction—Resonant Beam Techniques," *Univ. of Dayton Res. Inst. Vibration Damping Short Course Notes,* Nov. 1979.

[2] Henderson, J.P., "Layered Damping Treatment Design," *Univ. of Dayton Res. Inst. Vibration Damping Short Course Notes,* Nov. 1979.

[3] Skelton, R.E., "Large Space System Control Technology Model Order Reduction Study," in NASA Conference Pub. 2118, 1980.

[4] Papoulis, A., *The Fourier Integral and Its Applications,* McGraw-Hill, New York, 1962.

[5] Ungar, E.E. and Kerwin, E.M., Jr., "Loss Factors of Viscoelastic Systems in Terms of Energy Concepts," *Journal of the Acoustical Society of America,* Vol. 34, July 1962, pp. 954-957.

[6] Ungar, E.E., "The Status of Engineering Knowledge Concerning the Damping of Built-Up Structures," *Journal of Sound and Vibration,* Vol. 26, No. 1, 1973, pp. 141-154.

[7] Snowdon, J.C., *Vibration and Shock in Damped Mechanical Systems,* Wiley, New York, 1968.